International Journal of Education and Science Research Review
ISSN 2348-6457
Volume-1, Issue-4 August- 2014
www.ijesrr.org Email- editor@ijesrr.org

Development of an Inventory Model with Variable Demand Rate during Stock out Period

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ABSTRACT
In this paper an inventory model is developed in which shortages are allowed and partially backlogged. The backlogging rate is taken to be inversely proportional to the waiting time for the next replenishment. Demand follows power pattern. When fresh and new items arrive in stock they begin to decay after a fixed time interval called the life period of items. Here this realistic concept of non-instantaneous deterioration is taken into consideration with random rate of deterioration. Some special cases of the model also have been discussed. We can further extend this model with different demand rates and different backlogging rates.

INTRODUCTION

Most of the researchers have assumed that as soon as the items arrive in stock, they begin to deteriorate at once, but for many items this is not true. In practice when most of the items arrive in stock they are fresh and new and they begin to decay after a fixed time interval called life-period of items. Such deterioration is known as non-instantaneous deterioration. For fashionable commodities and other products with a short life cycle, the willingness of a customer to wait for backlogging during a shortage period is declining with the length of waiting time. Hence the longer the waiting time, the smaller the backlogging rate. Therefore the backlogging rate should be a variable and must be dependent on the waiting time for the next replenishment .It is important to control and maintain the inventories of deteriorating items for the modern corporation. In general, deterioration is defined as damage of any kind such that the item can not be used for its original purpose. Many Inventory models were developed for the static environment where the product's demand rate is assumed to be constant over the planning horizon. However, in a realistic product life cycle, demand is increasing with time during the growth phase. The effect of deterioration is very important in inventory systems. Food items, pharmaceuticals and radioactive substances are example of items in which sufficient deterioration can take place during the normal storage period of the units and consequently this loss must be taken into account when analyzing the system. In most of the literature deterioration is taken either constant or time dependent only. In real life it can be observed that deterioration depends upon many parameters. Since deterioration of an item depends upon the fluctuation of humidity, temperature, storage conditions etc. It would be more reasonable and realistic if we assume the deterioration function to depend upon one more parameter in addition to time, which ranges over a space in which some suitable probability density function is defined.
A power demand pattern inventory model for deteriorating items was developed by **Dutta** and **Pal** (1988). **Mandal** and **Phaujdar** (1989), **Pal et. al.** (1993) and **Giri et. al.** (1996) presented inventory models for perishable items with stock dependent demand rate.

When the shortages occur some customers are willing to wait for backorder and other would turn to buy from the other sellers. An optimal pricing and lot-sizing model under conditions of perish ability and partial backlogging was developed by **Abad** (1996). **Papachristos** and **Skouri** (2000), **Goyal** and **Giri** (2001) and **Yang et. al.** (2003) discussed inventory models for decaying items with partial backlogging. A comparison among various partial backlogging inventory models was given by **Yang** (2005) on the basis of maximum profit. In the presented chapter power demand pattern is taken with the concept of lifetime of items. Shortages are allowed and partially backlogged. In the present work an inventory model is developed in which shortages are allowed and partially backlogged. The backlogging rate is taken to be inversely proportional to the waiting time for the next replenishment. Demand follows power pattern. When fresh and new items arrive in stock they begin to decay after a fixed time interval called the life period of items. Here this realistic concept of non-instantaneous deterioration is taken into consideration with random rate of deterioration. Some special cases of the model also have been discussed.

NOTATIONS

(i) I (t) is the inventory level at time t, $t \geq 0$.

(ii) C, C_1, C_2, C_d, I denote the set up cost for each replenishment, inventory carrying cost per unit time, shortage cost for backlogged items, deterioration cost per unit, the unit cost of lost sales respectively. All of the cost parameters are positive constants.

(iii) μ is the life time of items.

(iv) T is the planning horizon.

(v) $\theta(t)$ is the variable deterioration rate s.t. $\vartheta(t) = \theta(\rho).t$, $0 < \theta(\rho) << 1$

(vi) t_1 is the time at which shortage starts.

(vii) S is the initial inventory after fulfilling backorders.

(viii) $\left(dt^{(1-n)/n}\right)/nT^{1/n}$ is the demand rate at time t.

(ix) K is the total average cost of the system.

ASSUMPTIONS

(i) The replenishment occurs instantaneously at an infinite rate.

(ii) No replacement or repair of deteriorated items is made during a given cycle.

(iii) A single item is considered over the prescribed period T units of time, which is subject to variable deterioration rate.

(iv) Lead time is zero.

(v) Shortages are allowed and backlogging rate is $\dfrac{\left(dt^{(1-n)/n}\right)/nT^{1/n}}{1+\alpha(T-t)}$, when inventory is in shortage. The backlogging parameter α is positive constant and $0<\alpha<<1$.

(vi) Power demand pattern is used

MATHEMATICAL FORMULATION AND ANALYSIS OF THE SYSTEM

Let Q be the total amount of inventory produced or purchased at the beginning of each cycle and after fulfilling backorders let us assume we get an amount S(>0) as initial inventory. During the period $(0,\mu)$ the inventory level gradually diminishes due to market demand only. After life time deterioration can take place, therefore during the period (μ, t_1) the inventory level decreases due to the market demand and deterioration of items and falls to zero at time t_1. The period (t_1,T) is the period of shortage, which is partially backlogged.

The differential equation governing the inventory level I(t) at any time t during the cycle (0,T) are such as

$$I'(t) = -\frac{dt^{(1-n)/n}}{nT^{1/n}}, \quad 0 \leq t \leq \mu \qquad \text{.....(1)}$$

$$I'(t)+\theta(t)\,I(t)=-\frac{dt^{(1-n)/n}}{nT^{1/n}},\quad \mu\le t\le t_1 \qquad \text{.....(2)}$$

and $$I'(t)=-\frac{dt^{(1-n)/n}}{nT^{1/n}\{1+\alpha(T-t)\}},\quad t_1\le t\le T \qquad \text{.....(3)}$$

The boundary conditions are

I(t)=S when t=0 .....(4)

and I(t)=0, when $t = t_1$ .....(5)

The solution of equation (1) Using boundary condition I(0)=S, is given by

$$I(t)=S-\frac{dt^{1/n}}{T^{1/n}},\quad 0\le t\le \mu \qquad \text{.....(6)}$$

At t = μ, inventory level becomes

$$I(\mu)=S-\frac{d\mu^{1/n}}{T^{1/n}} \qquad \text{.....(7)}$$

Solution of equation (2) using boundary condition at t = μ, is given as

$$I(t)\,e^{\theta(\rho)t^2/2}=$$

$$\frac{d}{T^{1/n}}\left[\left(\mu^{1/n}-t^{1/n}\right)+\frac{\theta(\rho)}{2(2n+1)}\left(\mu^{(2n+1)/n}-t^{(2n+1)/n}\right)\right]$$

$$+\left(S-\frac{d\mu^{1/n}}{T^{1/n}}\right)e^{\frac{\theta(\rho)\mu^2}{2}} \qquad \text{.....(8)}$$

By applying boundary condition (5), we can obtain the value of S as

$$S=\frac{d\mu^{1/n}}{T^{1/n}}+\frac{d}{T^{1/n}}$$

$$\left[\left(t_1^{\,1/n}-\mu^{1/n}\right)+\frac{\theta(\rho)}{2(2n+1)}\left(t_1^{\,(2n+1)/n}-\mu^{(2n+1)/n}\right)\right]e^{-\theta(\rho)\mu^2/2}$$

.....(9)

Using value of S in equation (8) We can get the solution of equation (2) as

$$I(t)=\frac{d}{T^{1/n}}$$

$$\left[\left(1-\frac{\theta(\rho)\,t^2}{2}\right)\left(t_1^{\,1/n}-t^{1/n}\right)+\frac{\theta(\rho)}{2(2n+1)}\right.$$

$$\left.\left(t_1^{\,(2n+1)/n}-t^{(2n+1)/n}\right)\right]$$

$$\mu\le t\le t_1 \qquad \text{.....(10)}$$

Solution of equation (3) by applying boundary condition is given by

$$I(t)=\frac{d}{T^{1/n}}\Big[(1-\alpha T)\ \left(t_1^{1/n}-t^{1/n}\right)$$
$$+\frac{\alpha}{n+1}\left(t_1^{(n+1)/n}-t^{(n+1)/n}\right)\Big],\ t_1\le t\le T \qquad ..(11)$$

During period (0,T) total number of units holding (I_H) can be obtained as

$$I_H=\int_0^{\mu}I(t)dt+\int_{\mu}^{t_1}I(t)dt$$
$$=\left(S\mu-\frac{d}{T^{1/n}}\frac{n}{n+1}\mu^{(n+1)/n}\right)+$$
$$\frac{d}{T^{1/n}}\Big[t_1^{1/n}(t_1-\mu)-\frac{n}{n+1}\left(t_1^{(n+1)/n}-\mu^{(n+1)/n}\right)$$
$$-\frac{\theta(\rho)}{6}t_1^{1/n}\left(t_1^3-\mu^3\right)+$$
$$\frac{\theta(\rho)}{2}\frac{n}{(3n+1)}\left(t_1^{(3n+1)/n}-\mu^{(3n+1)/n}\right)$$
$$+\frac{\theta(\rho)}{2(2n+1)}$$
$$\left\{t_1^{(2n+1)/n}(t_1-\mu)-\frac{n}{3n+1}\left(t_1^{(3n+1)/n}-\mu^{(3n+1)/n}\right)\right\}\Big]$$
$$=\frac{d}{T^{1/n}}\frac{\theta(\rho)}{3}\mu^3t_1^{1/n}+\frac{\theta(\rho)}{2}\mu^{(3n+1)/n}-\frac{\theta}{6}t_1^{(3n+1)/n}$$
$$+\frac{\theta(\rho)n}{2(3n+1)}\left(t_1^{(3n+1)/n}-\mu^{(3n+1)/n}\right)+$$
$$\frac{\theta(\rho)}{2(2n+1)}\Big\{t_1^{(3n+1)/n}-\mu^{(3n+1)/n}$$
$$+\frac{n}{3n+1}\left(\mu^{(3n+1)/n}-t_1^{(3n+1)/n}\right)\Big\}\Big]$$
$$=\frac{d}{T^{1/n}}\Big[\frac{1}{n+1}t_1^{(n+1)/n}-\frac{\theta(\rho)}{3}\mu^3t_1^{1/n}$$
$$+\frac{\theta(\rho)}{2}\left(\mu^{(3n+1)/n}-\frac{1}{3}t_1^{(3n+1)/n}\right)$$
$$+\frac{\theta(\rho)}{2}\frac{(n+1)}{(3n+1)}\left(t_1^{(3n+1)/n}-\mu^{(3n+1)/n}\right)\Big]$$
$$\Rightarrow I_H=\frac{d}{T^{1/n}}\Big[\frac{1}{n+1}t_1^{(3n+1)/n}-\frac{\theta(\rho)}{3}\mu^3t_1^{1/n}$$

$$+\frac{\theta(\rho)}{(3n+1)}\left(n\mu^{(3n+1)/n}+\frac{1}{3}t_1^{(3n+1)/n}\right)\Bigg]$$
..... (12)

Total amount of deteriorated units (I_D) during the period (0,T) is given by

$$I_D=\int_{\mu}^{t_1}\theta(t)\ I(t)\,dt$$

$$\Rightarrow I_D=\frac{d\theta(\rho)}{T^{1/n}}\left[\frac{1}{2}\left(t_1^{(2n+1)/n}-\mu^2\ t_1^{1/n}\right)\right.$$
$$\left.-\frac{n}{2n+1}\left(t_1^{(2n+1)/n}-\mu^{(2n+1)/n}\right)\right] \quad ..... (13)$$

Total amount of shortage units (I_S) during the period (0,T) is given as

$$I_S=-\int_{t_1}^{T}I(t)\,dt$$

$$I_S=\frac{d}{T^{1/n}}\left[\frac{n}{n+1}T^{(n+1)/n}+\frac{(1-2\alpha T)}{n+1}t_1^{(n+1)/n}\right.$$
$$\left.+(\alpha T-1)t_1^{1/n}.T-\frac{2\alpha n^2}{(2n+1)(n+1)}T^{(2n+1)/n}+\frac{\alpha}{2n+1}t_1^{(2n+1)/n}\right]$$
..... (7.14)

Total amount of lost sales (I_L) during the period (0,T) is given by

$$I_L=\int_{t_1}^{T}\left\{1-\frac{1}{1+\alpha(T-t)}\right\}\frac{dt^{(1-n)/n}}{nT^{1/n}}dt$$

$$=\frac{d\alpha}{nT^{1/n}}\int_{t_0}^{T}\left(t^{(1-n)/n}T-t^{1/n}\right)dt$$

$$\Rightarrow I_L=\frac{d\alpha}{T^{1/n}}\left[\frac{n}{n+1}T^{(n+1)/n}-t_1^{1/n}.T+\frac{1}{n+1}t_1^{(n+1)/n}\right] \quad ..... (15)$$

Total average cost of the system per unit time is given by

$$K=\frac{1}{T}\left[C+C_1I_H+C_dI_D+C_2I_S+I.I_L\right]$$

$$=\frac{C}{T}+\frac{d}{T^{(n+1)/n}}\left[C_1\left\{\frac{1}{n+1}t_1^{(n+1)/n}-\frac{\theta(\rho)}{3}\mu^3t_1^{1/n}\right.\right.$$
$$\left.+\frac{\theta(\rho)}{(3n+1)}\left(n\mu^{(3n+1)/n}+\frac{1}{3}t_1^{(3n+1)/n}\right)\right\}$$
$$+C_d\theta(\rho)\left\{\frac{1}{2}\left(t_1^{(2n+1)/n}-\mu^2t_1^{1/n}\right)\right.$$

$$-\frac{n}{2n+1}\left(t_1^{(2n+1)/n}-\mu^{(2n+1)/n}\right)\Bigg\}$$

$$+C_2\Bigg\{\frac{n}{n+1}T^{(n+1)/n}$$

$$+\frac{(1-2\alpha T)}{n+1}t_1^{(n+1)/n}+(\alpha T-1)t_1^{1/n}.T-\frac{2\alpha n^2}{(2n+1)(n+1)}T^{(2n+1)/n}$$

$$+\frac{\alpha}{(2n+1)}t_1^{(2n+1)/n}\Bigg\}+I\alpha\Bigg\{\frac{n}{n+1}T^{(n+1)/n}-t_1^{1/n}T+\frac{1}{n+1}.t_1^{(n+1)/n}\Bigg\}\Bigg]$$

.. (16)

APPROXIMATION SOLUTION PROCEDURE

To minimize total average cost per unit time (K), the optimal values of t_1 and T can be obtained by solving the following equations simultaneously.

$$\frac{\partial K}{\partial t_1}=0 \qquad ..... (17)$$

and $$\frac{\partial K}{\partial T}=0 \qquad ..... (18)$$

Provided they satisfy the following conditions.

$$\frac{\partial^2 K}{\partial t_1^2}>0 \quad , \quad \frac{\partial^2 K}{\partial T^2}>0$$

and $$\left(\frac{\partial^2 K}{\partial t_1^2}\right)\left(\frac{\partial^2 K}{\partial T^2}\right)-\left(\frac{\partial^2 K}{\partial t_1 \partial T}\right)^2>0$$

Equations (17) and (18) are equivalent to-

$$C_1\left[t_1+\frac{\theta(\rho)}{3}\left(t_1^3-\mu^3\right)\right]+C_d\frac{\theta(\rho)}{2}\left(t_1^2-\mu^2\right)$$

$$+(t_1-T)\left[C_2\{1+\alpha(t_1-T)\}+I\alpha\right]=0 \qquad ..... (19)$$

and $$CT^{1/n}+\frac{(n+1)}{n}d$$

$$\Bigg[C_1\Bigg\{\frac{1}{n+1}t_1^{(n+1)/n}-\frac{\theta(\rho)}{3}\mu^3 t_1^{1/n}+\frac{\theta(\rho)}{3n+1}$$

$$\left(n\mu^{(3n+1)/n}+\frac{1}{3}t_1^{(3n+1)/n}\right)\Bigg\}+$$

$$C_d\theta(\rho)\Bigg\{\frac{1}{2}\left(t_1^{(2n+1)/n}-\mu^2 t_1^{1/n}\right)$$

$$-\frac{n}{2n+1}\left(t_1^{(2n+1)/n}-\mu^{(2n+1)/n}\right)\Bigg\}+C_2\left\{\frac{n}{n+1}T^{(n+1)/n}\right.$$

$$+\frac{(1-2\alpha T)}{n+1}t_1^{(n+1)/n}+(\alpha T-1)\,t_1^{1/n}T-\frac{2\alpha n^2}{(2n+1)(n+1)}T^{(2n+1)/n}$$

$$\left.+\frac{\alpha}{(2n+1)}t_1^{(2n+1)/n}\right\}+I\alpha\left\{\frac{n}{n+1}T^{(n+1)/n}-t_1^{1/n}T+\frac{1}{n+1}.t_1^{(n+1)/n}\right\}\Bigg]$$

$$-dT\left[(C_2+I\alpha)T^{1/n}+\{C_2(2\alpha T-1)-I\alpha\}t_1^{1/n}-\frac{2\alpha C_2}{n+1}\left\{nT^{(n+1)/n}+t_1^{(n+1)/n}\right\}\right]=0$$

….. (20)

Equations (19) and (20) can be solved by some suitable computational methods with the help of computer to obtain the optimal values of t_1 and T. With the use of these optimal values equation (16) provides minimum total average cost per unit time of the system in consideration.

SPECIAL CASES

CASE1: If the planning horizon is finite, then the cost function K in the presented model will convert to the function, which depends on t_1, and condition for minimization of the total average cost per unit time becomes

$$\frac{dK}{dt_1}=0$$

….. .21)

The value of t_1 obtained by equation (21) is the optimal value of t_1 provided it satisfies the sufficient condition.

$$\frac{d^2K}{dt_1^2}>0$$

…..(22)

The optimal values of t_1 obtained from equation (21) will give the minimum total average cost per unit time.

CASE2: If μ=0, then the discussed model reduces to the model without lifetime.

CONCLUSION

In this chapter an inventory model is developed with variable rate of deterioration and power demand pattern. The backlogging rate is taken as time dependent. Cost minimization technique is used to get the approximate expressions for total cost and other parameters. Two special cases of the model are also discussed. We can further extend this model with different demand rates and different backlogging rates.

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